



The Philosophy of Logical Analysis
(Chapter XXXI of "A History of Western Philosophy")

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In philosophy ever since the time of Pythagoras there has been an opposition between the men whose thought was mainly inspired by mathematics and those who were more influenced by the empirical sciences. Plato, Thomas Aquinas, Spinoza, and Kant belong to what may be called the mathematical party; Democritus, Aristotle, and the modern empiricist from Locke onwards, belong to the opposite party. In our day a school of philosophy has arisen which sets to work to eliminate Pythagoreanism from the principles of mathematics, and to combine empiricism with an interest in the deductive parts of human knowledge. The aims of this school are less spectacular than those of most philosophers in the past, but some of its achievements are as solid as those of the men of science.

The origin of this philosophy is in the achievements of mathematicians who set to work to purge their subject of fallacies and slipshod reasoning. The great mathematicians of the seventeenth century were optimistic and anxious for quick results; consequently they left the foundations of analytical geometry and the infinitesimal calculus insecure. Leibniz believed in actual infinitesimals, but although this belief suited his metaphysics it has no sound basis in mathematics. Weierstrass, soon after the middle of the nineteenth century, showed how to establish the calculus without infinitesimals, and thus at last made it logically secure. Next came Georg Cantor, who developed the theory of continuity and infinite number. "Continuity" had been, until he defined it, a vague word, convenient for philosophers like Hegel, who wished to introduce metaphysical muddles into mathematics. Cantor gave a precise significance to the word, and showed that continuity, as he defined it, was the concept needed by mathematicians and physicist. By this means a great deal of mysticism, such as that of Bergson, was rendered antiquated.

Cantor also overcame the long-standing logical puzzles about infinite number. Take the series of whole numbers from 1 onwards; how many of them are there? Clearly the number is not finite. Up to a thousand, there are a thousand numbers; up to million, a million. Whatever finite number you mention, there are evidently more numbers than that, because from 1 up to the number in question there are just that number of numbers, and then there are others that are greater. The number of finite whole numbers must, therefore, be an infinite number. But now comes a curious fact: The number of even numbers must be the same as the number of all whole numbers. Consider two rows:

1, 2, 3, 4, 5, 6,

2, 4, 6, 8, 10, 12,

There is one entry in the lower row for every one in the top row; therefore the number of terms in the two rows must be the same, although the lower row consists of only half the terms in the top row. Leibniz, who noticed this, thought it a contradiction, and concluded, though there are infinite collections, there are no infinite numbers. Georg Cantor, on the contrary, boldly denied that it is a contradiction. He was right; it is only an oddity.

Georg Cantor defined an "infinite" collection as one which has parts containing as many terms as the whole collection contains. On this basis he was able to build up a most interesting mathematical theory of infinite numbers, thereby

taking into realm of exact logic a whole region formerly given over to mysticism and confusion.

The next man of importance was Frege, who published his first work in 1879, and his definition of "number" in 1884; but in spite of the epoch-making nature of his discoveries, he remained wholly without recognition until I drew attention to him in 1903. It is remarkable that, before Frege, every definition of number that had been suggested contained elementary logical blunders. It was customary to identify "number" with "plurality". But an instance of number is a particular number, say 3, and an instance of 3 is a particular triad. The triad is a plurality, but the class of all triads - which Frege identified with the number 3 - is a plurality of pluralities, and number in general, of which 3 is an instance, is a plurality of pluralities of pluralities. The elementary grammatical mistake of confounding this with the simple plurality of a given triad made the whole philosophy of number, before Frege, a tissue of nonsense in the strictest sense of the term "nonsense".

From Frege's work it followed that arithmetic, and pure mathematics generally, is nothing but a prolongations of deductive logic. This disproved Kant's theory that arithmetical propositions are "synthetic" and involve reference to time. The development of pure mathematics from logic was set forth in detail in *Principia Mathematica*, by Whitehead and myself.

It gradually became clear that a great part of philosophy can be reduced to something that may be called "syntax", though the word has to be used in a somewhat wider sense than has hitherto been customary. Some men, notably Carnap, have advanced the theory that all philosophical problems are really syntactical, and that, when errors in syntax are avoided, a philosophical problem is thereby either solved or shown to be insoluble. I think this is an overstatement, but there can be no doubt that the utility of philosophical syntax in relation to traditional problems is very great.

I will illustrate its utility by a brief explanation of what is called the theory of descriptions. By "description" I mean a phrase such as "The present President of the United States", in which a person or thing is designated, not by name, but by some property which is supposed or known to be peculiar to him or it. Such phrases had given a lot of trouble. Suppose I say "The golden mountain does not exist" and suppose you ask "What is it that does not exist?" It would seem that, if I say "It is the golden mountain", I am attributing some sort of existence to it. Obviously I am not making the same statement as if I said, "The round square does not exist". This seemed to imply that the golden mountain is one thing and the round square is another, although neither exists. The theory of descriptions was designed to meet this and other difficulties.

According to this theory, when a statement containing a phrase of the form "the so-and-so" is rightly analyzed, the phrase "the so-and-so" disappears. For example, take the statement "Scott was the author of *Waverley*". The theory interprets this statement as saying:

"One and the only one man wrote *Waverley*, and that man was Scott". Or, more fully:

"There is an entity c such that the statement 'x wrote *Waverley*' is true if x is c and false otherwise; moreover c is Scott".

The first part of this, before the word “moreover”, is defined as meaning: “The author of *Waverley* exist (or existed or will exist)”. Thus “The golden mountain does not exist” means:

“There is no entity c such that ‘ x is golden and mountainous’ is true when x is c , but not otherwise”.

With this definition the puzzle as to what is meant when we say “The golden mountain does not exist” disappears.

“Existence”, according to this theory, can only be asserted of descriptions. We can say “The author of *Waverley* exists”, but to say “Scott exists” is bad grammar, or rather bad syntax. This clears up two millennia of muddle-headedness about “existence”, beginning with Plato’s *Theaters*.

One result of the work we have been considering is to dethrone mathematics from the lofty place that it has occupied since Pythagoras and Plato, and to destroy the presumption against empiricism which has been derived from it. Mathematical knowledge, if it is true, is not obtained by induction from experience; our reason for believing that 2 and 2 are 4 is not that we have so often found, by observation, that one couple and another couple together make a quartet. In this sense, mathematical knowledge is still not empirical. But it is also not *a priori* knowledge about the world. It is, in fact, merely verbal knowledge. “3” means “2 + 1” and “4” means “3 + 1”. Hence it follows (though the proof is long) that “4” means the same as “2 + 2”. Thus mathematical knowledge ceases to be mysterious. It is all of the same nature as the “great truth” that there are three feet in a yard.

Physics, as well as pure mathematics, has supplied material for the philosophy of logical analysis. This has occurred especially through the theory of relativity and quantum mechanics.

What is important to the philosopher in the theory of relativity is the substitution of space-time for space and time. Common sense thinks of the physical world as composed of “things” which persist through a certain period of time and move in space. Philosophy and physics developed the notion of “thing” into that of “material substance”, and thought of material substance as consisting of particles, each very small, and each persisting throughout all time. Einstein substituted events for particles; each event had to each other a relation called “interval”, which could be analyzed in various ways into a time-element and space-element. The choice between these various ways was arbitrary and no one of them was theoretically preferable to any other. Given two elements A and B, in different regions, it might happen that according to one convention they were simultaneous, according to another A was earlier than B, and according to yet another B was earlier than A. No physical facts correspond to these different conventions.

From all this it seems to follow that events, not particles, must be the “stuff” of physics. What has been thought of as a particle will have to be thought of as a series of events. The series of events that replaces a particle has certain important physical properties, and therefore demands our attention; but it has no more substantiality than any other series of events that we might arbitrarily single out. Thus “matter” is not part of the ultimate material of the world, but merely a convenient way of collecting events into bundles.

Quantum theory reinforces this conclusion, but its chief philosophical importance is that it regards physical phenomena as possibly discontinuous. It suggests that, in an atom (interpreted as above), a certain state of affairs persists for a certain time, and then suddenly is replaced by a finitely different state of affairs. Continuity of motion, which had always been assumed, appears to have been a mere prejudice. The philosophy appropriate to quantum theory, however, has not yet been adequately developed. I suspect that it will demand even more radical departures from the traditional doctrine of space and time than those demanded by the theory of relativity.

While physics has been making matter less material, psychology has been making mind less mental. We had occasion in a former chapter to compare the association of ideas with the conditioned reflex. The latter, which has replaced the former, is much more physiological. (This is only one illustration; I do not wish to exaggerate the scope of the conditioned reflex). Thus from both ends physics and psychology have been approaching each other, and making more possible the doctrine of "neutral monism" suggested by William James's criticism of "consciousness". The distinction of mind and matter came into philosophy from religion, although, for a long time, it seemed to have valid grounds. I think that both mind and matter are merely convenient ways of grouping events. Some single events, I should admit, belong only to material groups, but others belong to both kinds of groups, and are therefore at once mental and material. This doctrine effects a great simplification in our picture of the structure of the world.

Modern Physics and physiology throw a new light upon the ancient problem of perception. If there is to be anything that can be called "perception", it must be in some degree an effect of the object perceived, and must more or less resemble the object if it is to be a source of knowledge of the object. The first requisite can only be fulfilled if there are causal chains which are, to a greater or less extent, independent of the rest of the world. According to physics, this is the case. Light-waves travel from the sun to the earth, and in doing so obey their own laws. This is only roughly true. Einstein has shown that light-rays are affected by gravitation. When they reach our atmosphere, they suffer refraction, and some are more scattered than others. When they reach a human eye, all sorts of things happen which would not happen elsewhere, ending up with what we call "seeing the sun". But although the sun of our visual experience is very different from the sun of the astronomer, it is still a source of knowledge as to the latter, because "seeing the sun" differs from "seeing the moon" in ways that are causally connected with the difference between the astronomer's sun and the astronomer's moon. What we can know of physical objects in this way, however, is only certain abstract properties of structure. We can know that the sun is round in a sense, though not quite the sense in which what we see is round; but we have no reason to suppose that it is bright or warm, because physics can account for its seeming so without supposing that it is so. Our knowledge of the physical world, therefore, is only abstract and mathematical.

Modern analytical empiricism, of which I have been giving an outline, differs from that of Locke, Berkeley, and Hume by its incorporation of mathematics and its development of powerful logical technique. It is thus able, in regard to certain

problems, to achieve definite answers, which have the quality of science rather than of philosophy. It has the advantage, as compared with the philosophies of the system-builders, of being able to tackle its problems one at a time, instead of having to invent at one stroke a block theory of the whole universe. Its methods, in this respect, resemble those of science. I have no doubt that, in so far as philosophical knowledge is possible, it is by such methods, many ancient problems are completely soluble.

There remains, however, a vast field, traditionally included in philosophy, where scientific methods are inadequate. This field includes ultimate questions of value; science alone, for example, cannot prove that it is bad to enjoy the infliction of cruelty. Whatever can be known, can be known by means of science; but things which are legitimately matters of feeling lie outside its province.

Philosophy, throughout its history, has consisted of two parts inharmoniously blended; on the one hand a theory as to the nature of the world, on the other an ethical or political doctrine as to the best way of living. The failure to separate these two with sufficient clarity has been a source of much confused thinking. Philosophers, from Plato to William James, have allowed their opinions as to the constitution of the universe to be influenced by the desire for edification: knowing, as they supposed, what beliefs would make men virtuous, they have invented arguments, often very sophisticated, to prove that these beliefs are true. For my part I reprobate this kind of bias, both on moral and on intellectual grounds. Morally, a philosopher who uses his professional competency for anything except a disinterested search for truth is guilty of a kind of treachery. And when he assumes, in advance of inquiry, that certain beliefs, whether true or false, are such as to promote good behavior, he is so limiting the scope of philosophical speculation as to make philosophy trivial; the true philosopher is prepared to examine *all* preconceptions. When any limits are placed, consciously or unconsciously, upon the pursuit of truth, philosophy becomes paralyzed by fear, and the ground is prepared for a government censorship punishing those who utter "dangerous thoughts" - in fact, the philosopher has already placed such a censorship over his own investigations.

Intellectually, the effect of mistaken moral considerations upon philosophy has been to impede progress to an extraordinary extent. I do not myself believe that philosophy can either prove or disprove the truth of religious dogmas, but ever since Plato most philosophers have considered it part of their business to produce "proofs" of immortality and the existence of God. They have found fault with the proofs of their predecessors - Saint Thomas rejected Saint Anselm's proofs, and Kant rejected Descartes' - but they have supplied new ones of their own. In order to make their proofs seem valid, they have had to falsify logic, to make mathematics mystical, and to pretend that deep-seated prejudices were heaven-sent intuitions.

All this is rejected by the philosophers who make logical analysis the main business of philosophy. They confess frankly that the human intellect is unable to find conclusive answers to many questions of profound importance to mankind, but they refuse to believe that there are some "higher" way of knowing, by which we can discover truths hidden from science and the intellect. For this renunciation they

have been rewarded by the discovery that many questions, formerly obscured by the fog of metaphysics, can be answered with precision, and by objective methods which introduce nothing of the philosopher's temperament except the desire to understand. Take such questions as: What is number? What are space and time? What is mind, and what is matter? I do not say that a method has been discovered by which, as in science, we can make successive approximations to the truth, in which each new stage results from an improvement, not a rejection, of what has gone before.

In the welter of conflicting fanaticisms, one of the few unifying forces is scientific truthfulness, by which I mean the habit of basing our beliefs upon observations and inferences as impersonal, and as much diverse of local and temperamental bias, as is possible for human beings. To have insisted upon the introduction of his virtue into philosophy, and to have invented a powerful method by which it can be rendered fruitful, are the chief merits of the philosophical school of which I am a member. The habit of careful veracity acquired in the practice of this philosophical method can be extended to the whole sphere of human activity, producing, wherever it exists, a lessening of fanaticism with an increasing capacity of sympathy and mutual understanding. In abandoning a part of its dogmatic pretensions, philosophy does not cease to suggest and inspire a way of life.

